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To: Professor Peter Fishburn
AT&T BELL LABORATORIES

Dear Peter,

I got your interim-update on the independence axiom today. It read like an exciting novel. Who-dun-it? This question had been with me for a long time, so it is exciting to see it tackled now. I think that laying down the findings of this enterprise, and publishing it somewhere, is worthwhile, even while it could be brief. The independence condition has turned out to be the most important idea in DMUR/DMUU. It took quite some correspondence and literature-checking to get this work done, and you did it all, in an excellent way, finding (almost) all people willing to give you their best information.

The origin of the independence condition has not been very heroic. It grew out of a shortcoming of von Neumann & Morgenstern, not a mathematical shortcoming, but an intuitive shortcoming (taking equivalence classes as primitives). It is a well-known and standard mathematical fact that, if one works with equivalence classes, and transfers operations to these, then one should check that it is immaterial for the operation which representatives from the equivalence classes are taken. Applying that general principle to the mixing operation given the independence condition. So if von Neumann would have said: 'I knew this condition but considered it too self-evident to write down', then that would be credible to me. The enjoyable letters of which you sent me copies suggest that this was obvious to Arrow (he credits discussions with others), and that Samuelson sensed it but did not fully get hold of it. Nash clearly indicates that he got his ideas from von Neumann & Morgenstern. Whether Marschak got his idea for the independence condition from the shortcoming of vNM, or independently, I do not know.

Well, the question now also raised by Suppes, is also interesting, the more so because the condition is a close relative. The history and origin of independence from consumer demand theory (separability) is as vague and intriguing. Here the condition is like: the rate of substitution between two commodities is independent of the level at which the other commodities are kept fixed. Loosely, this implies additive representability. Samuelson in his 1947 book, pp.174-180, already seemed to know this. Also Gorman wrote somewhere that he knew his famous result already in the

fifties. I guess economists may have known this as common knowledge long ago. Sono and Leontief are relevant references here.

What would you say if independence would be traced back to the old Greeks, and it was found that there was a term for the condition in Greek that had been lost in English? This is the case at least according to p.468 of Friedman & Savage (1952), in relation to the sure-thing principle:

"practically unique among maxims for wise action in the face of uncertainty, in the strength of its intuitive appeal. The principle is universally known and recognized; and the Greeks must surely have had a name for it, though current English seems not to".

Of course we shouldn't take this too seriously, it is just a funny citation.

Best wishes,

Peter Wakker